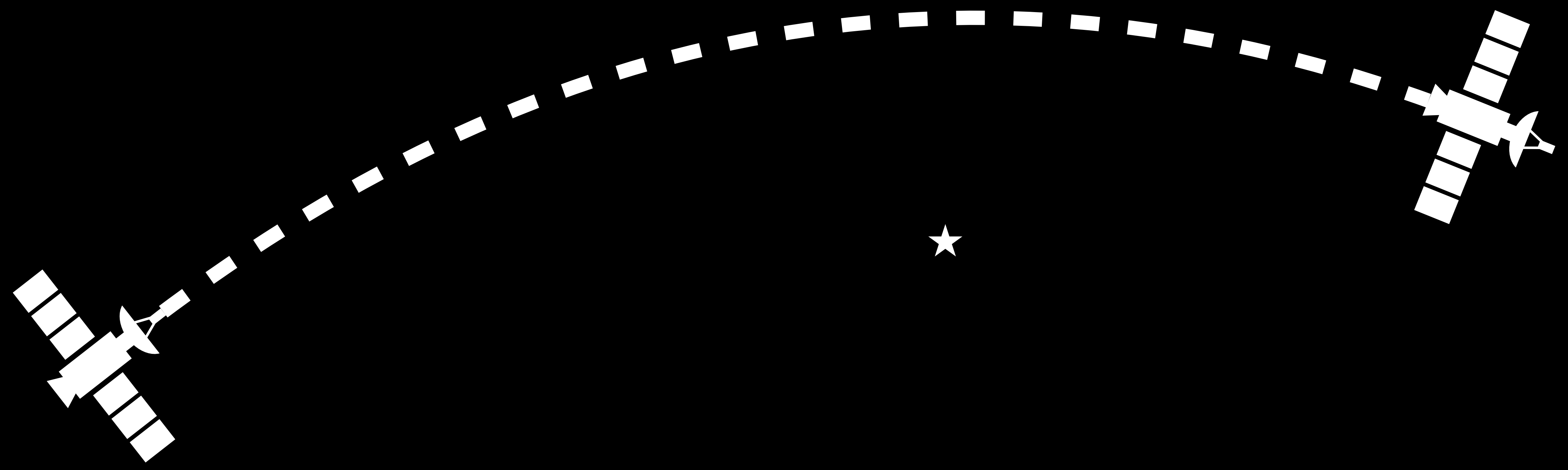


Model Predictive Control

Applied to Low Earth Orbit Satellites

Raaghav Thirumaligai

overview

Topics Covered

- ❖ Problem Setup
- ❖ Discretization
- ❖ MPC Formulations
- ❖ Results / Comparison to LQR
- ❖ Discussion & Comparison

Problem Setup

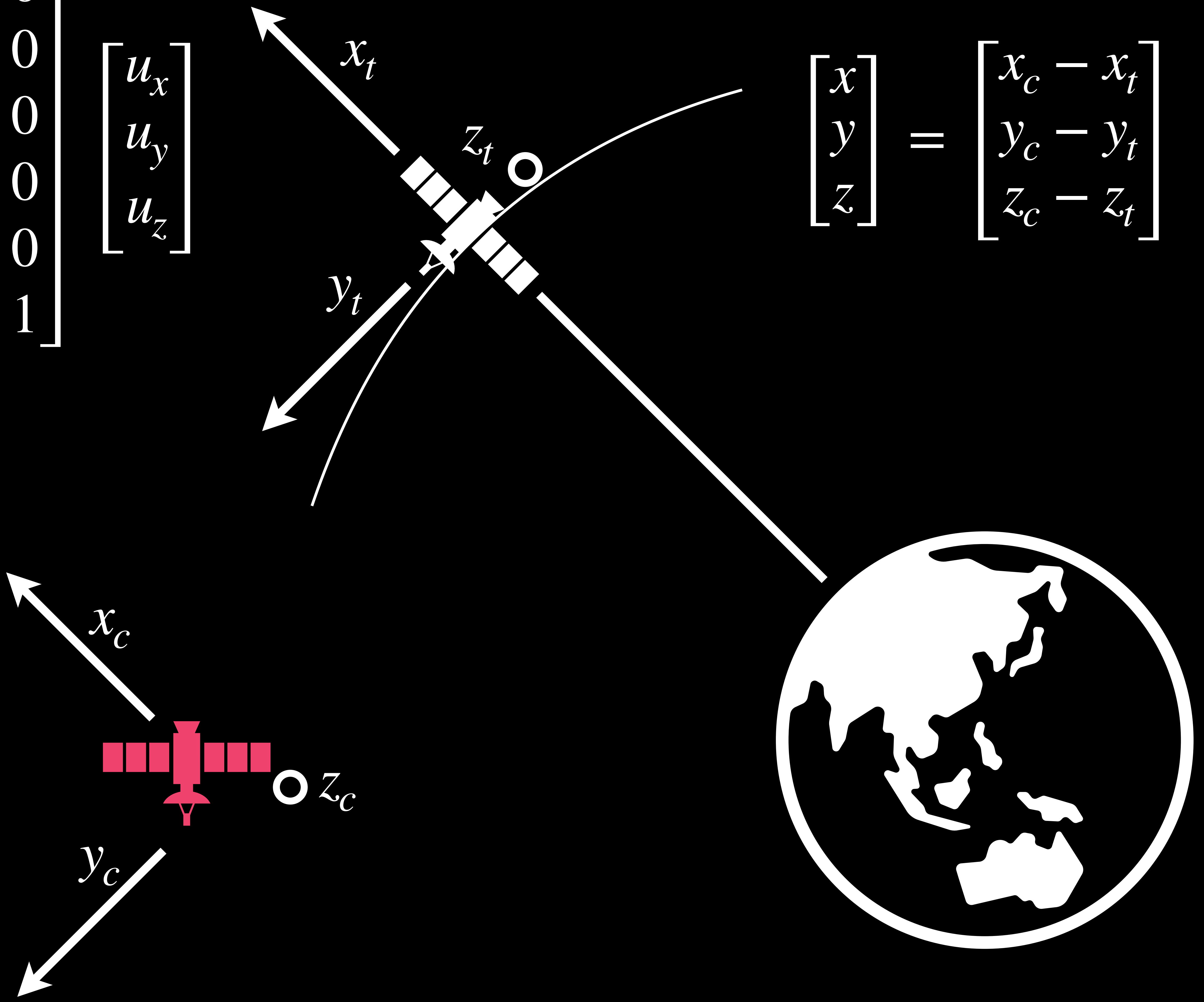
Hill - Clohessy - Wiltshire equations [2]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$n \approx 0.001$ Hz is the orbital period

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \text{radial relative position} \\ \text{along-track relative position} \\ \text{cross-track relative position} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_c - x_t \\ y_c - y_t \\ z_c - z_t \end{bmatrix}$$



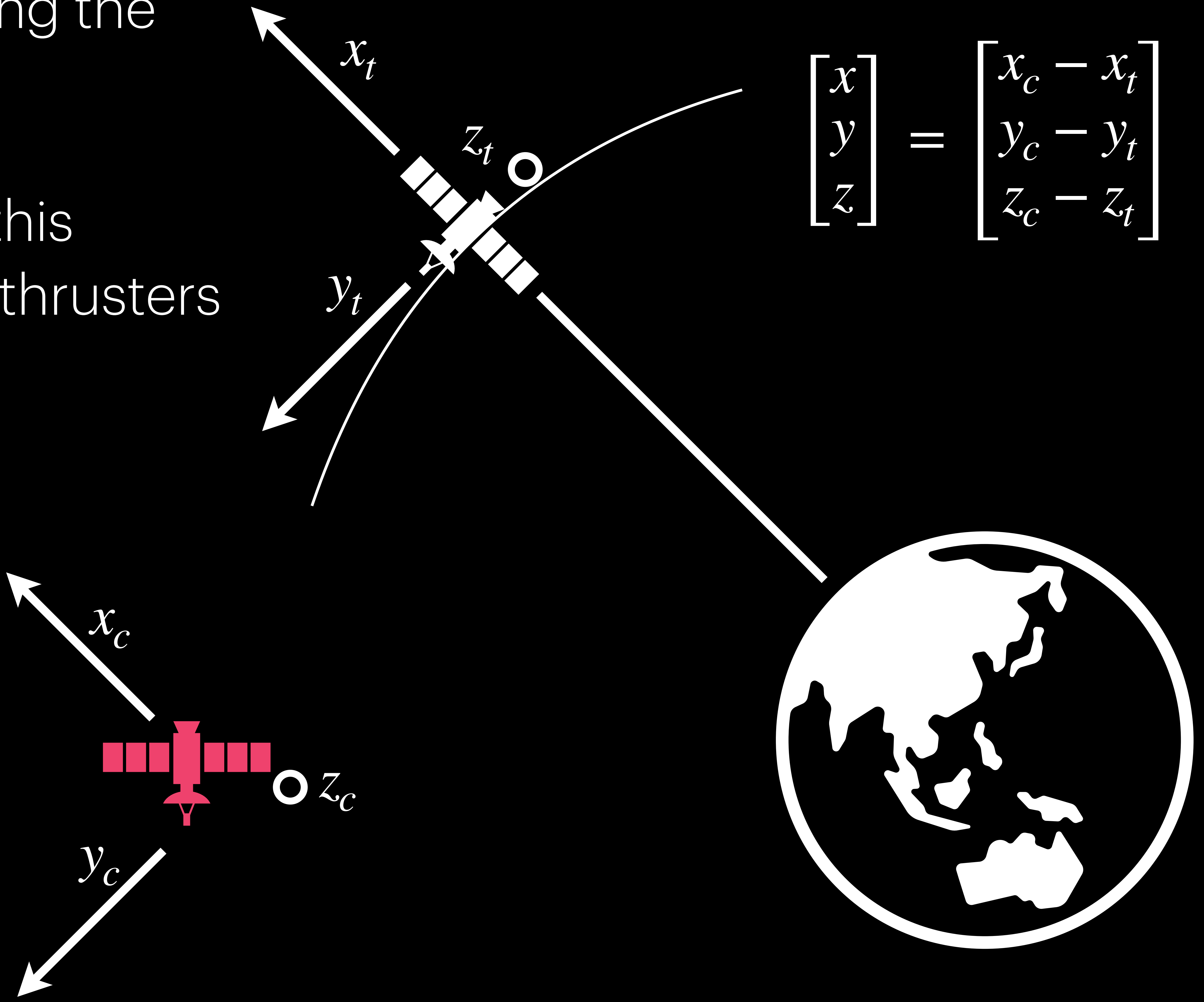
Problem Setup

Dynamics

- Driving the state to zero is equivalent to driving the chaser satellite to the target satellite

- *someone* has to map the input sequence in this orientation back to the directions of the real thrusters

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_c - x_t \\ y_c - y_t \\ z_c - z_t \end{bmatrix}$$

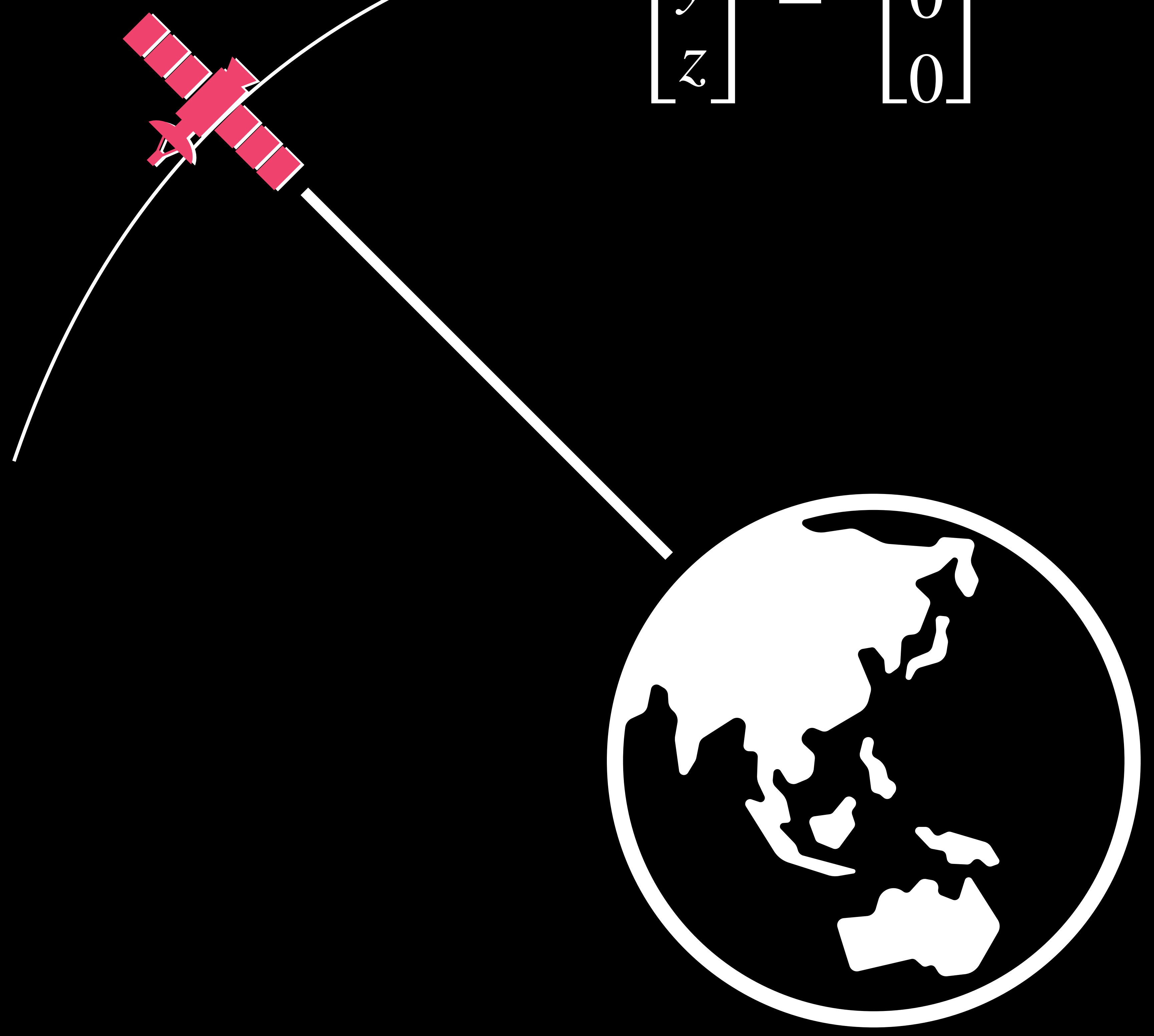
Problem Setup

Why MPC?

- We have natural input constraints[1] from what the thruster can provide

- LQR can probably be tuned to work but a reasonable effort by saturating inputs gives much worse performance (more on that later)

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \in [-0.5, 0.5]\ \frac{\text{mm}}{\text{s}^2}$$



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Discretization

Continuous to discrete via ZOH

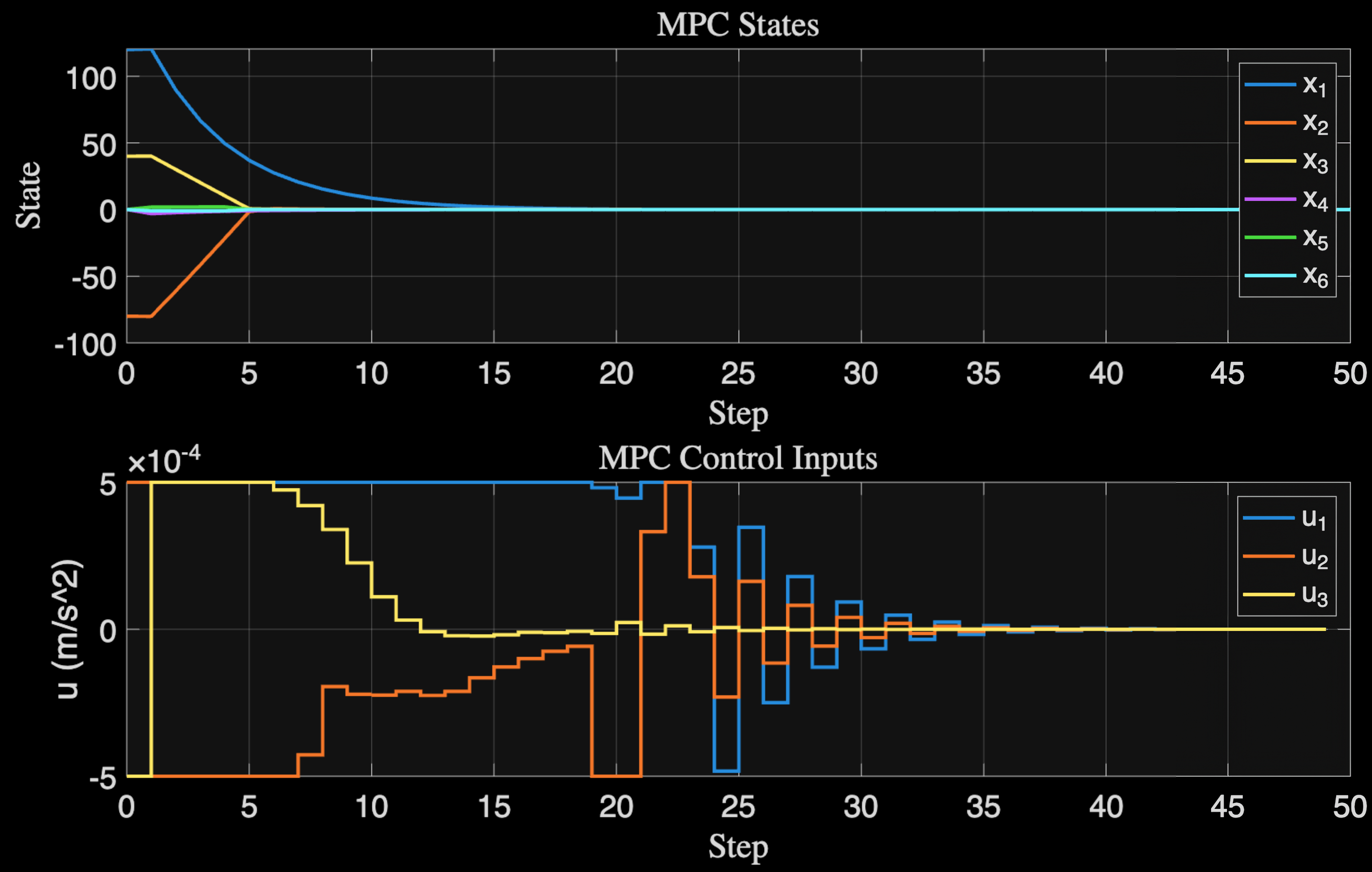
$$\left.\begin{aligned} A_d &= e^{A_c T_s} \\ B_d &= \int_0^{T_s} e^{A_c \tau} d\tau B_c \end{aligned}\right\} \Longrightarrow \text{expm}\left(\begin{bmatrix} A_c & B_c \\ 0 & 0 \end{bmatrix} \cdot T_s\right) = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}$$

- Using a zero order hold is fine since we have relatively slow dynamics
- $T_s = 10\,\mathrm{s}$ chosen s.t. $T_s \ll \frac{1}{n}$

MPC attempt v2

Small l_2 cost on input signal

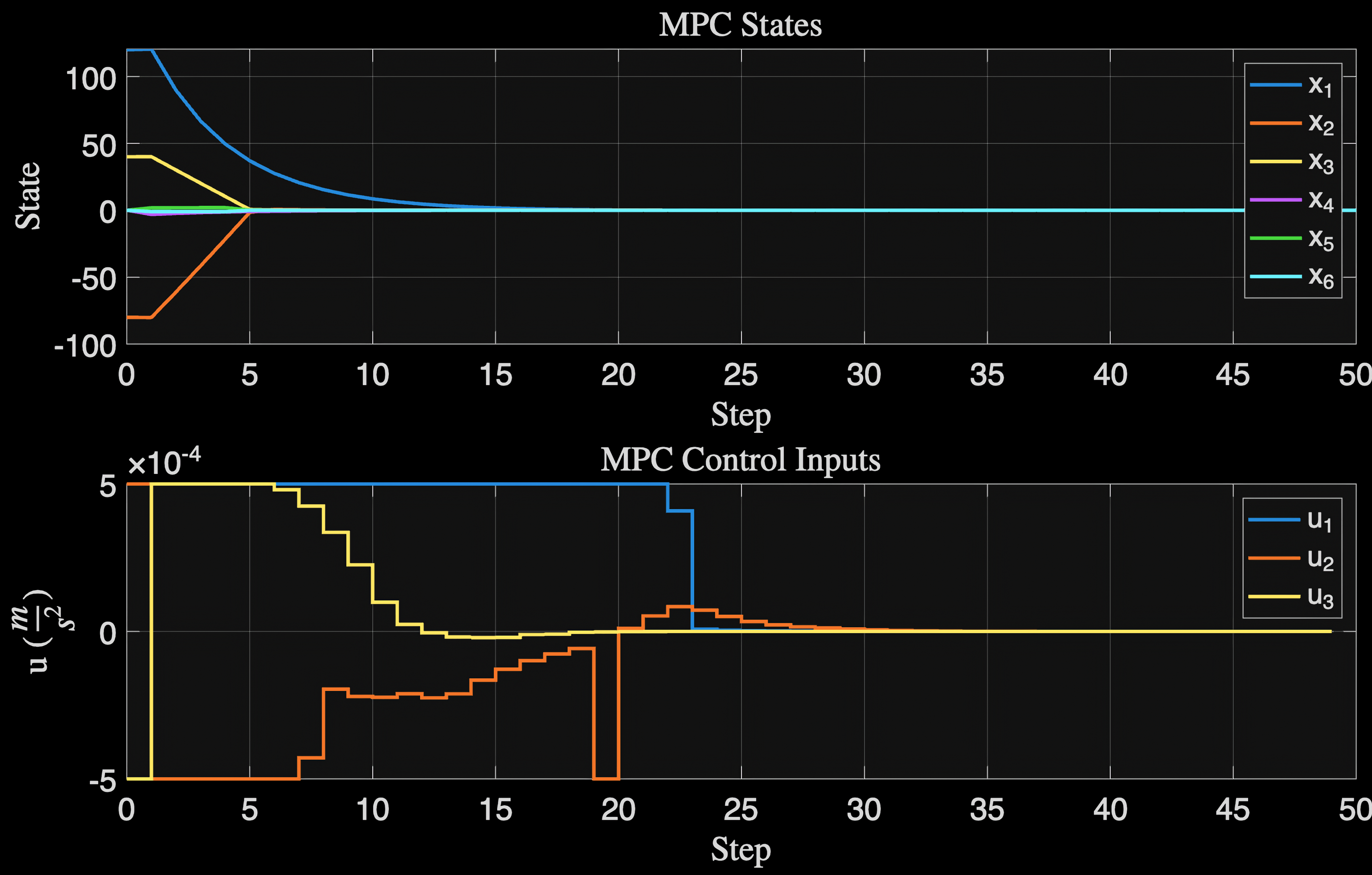
- Prediction horizon $N_t = 5$
- Terminal set constraint $x_{N+1} = 0$
- Stage cost: $l(x, u) := x'Qx + u'Ru$ with $R \ll Q$
- Terminal cost: $V_f(x) := x'Q_f x$ with $Q_f \approx Q$
- *Fuel Cost*: 0.026758



MPC attempt v1... *Economic MPC*

Large l_1 cost on input signal

- Augmenting the stage cost to directly penalize fuel expense rather than
- Stage cost: $l(x, u) := x'Qx + w \cdot \|u\|_1$ with $Q \ll w$
- Terminal cost: $V_f(x) := 0$
- *Fuel Cost*: 0.0227286
- THATS A 15% DECREASE!!



Its linear... what about LQR??

Its linear… what about LQR??

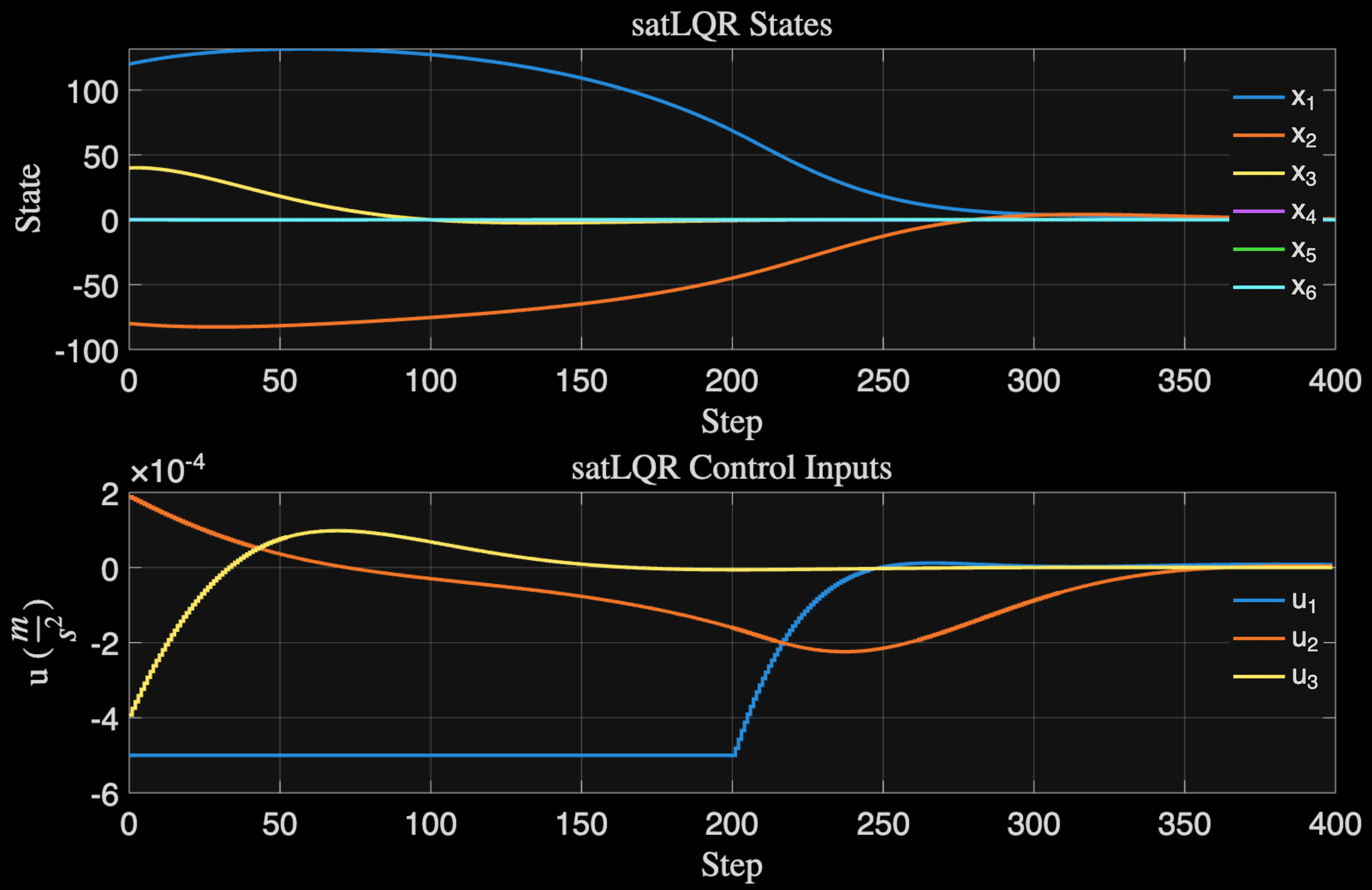
Show me that controller

- We already know that both MPC follow similar paths

- That convergence isn't so bad… wait…

- Simulation ran about 8x longer to become stable trajectories large for a long time

- This was not an easy controller to tune!



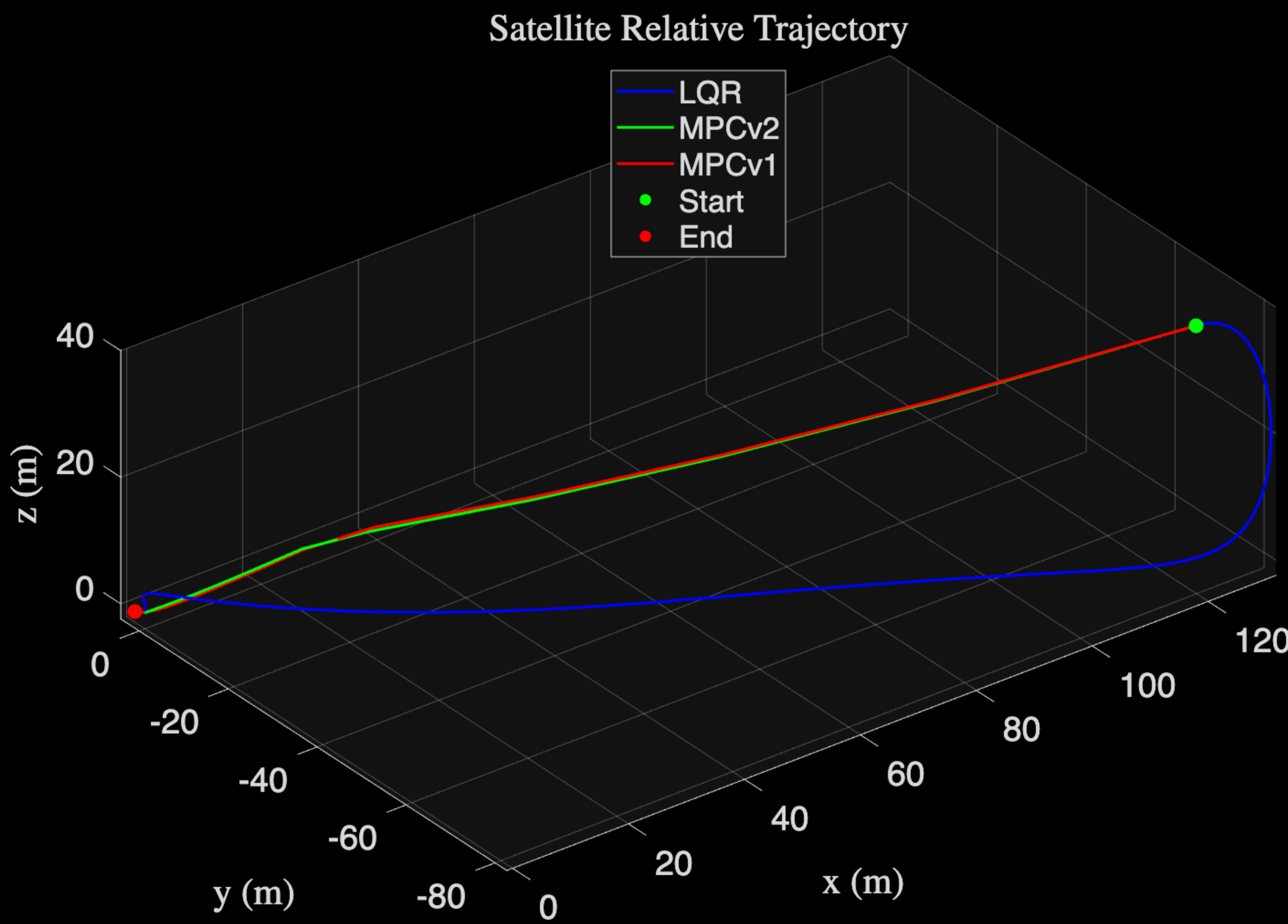
Simulation Results

Trajectories & Trade-offs

- We already know that both MPC follow similar paths

- The LQR eventually gets there but takes 8x longer for this IC

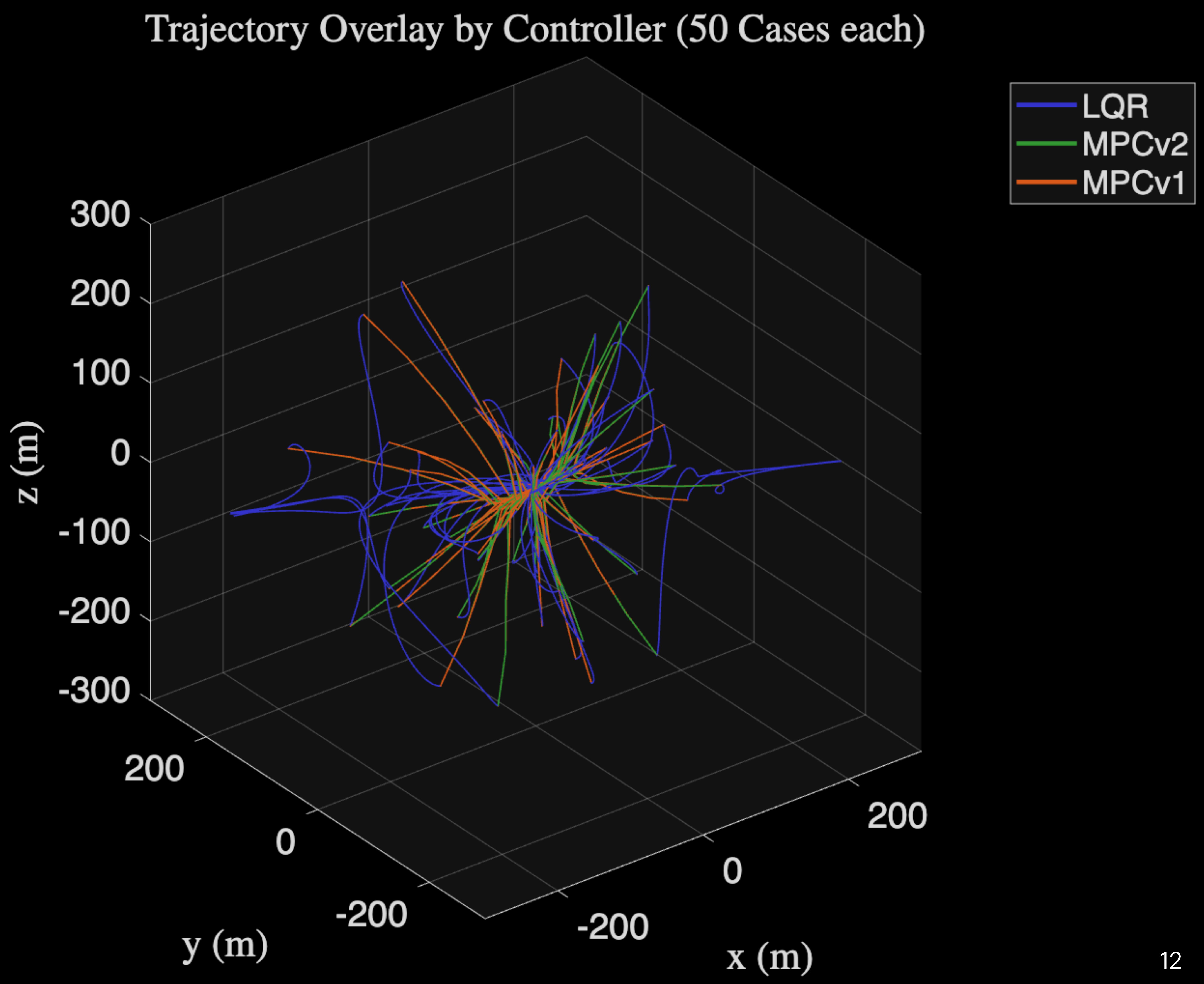
- OK so now one might wonder if **“BIG MPC”** is cherry-picking a difficult initial condition



Simulation Results

Trajectories & Trade-offs

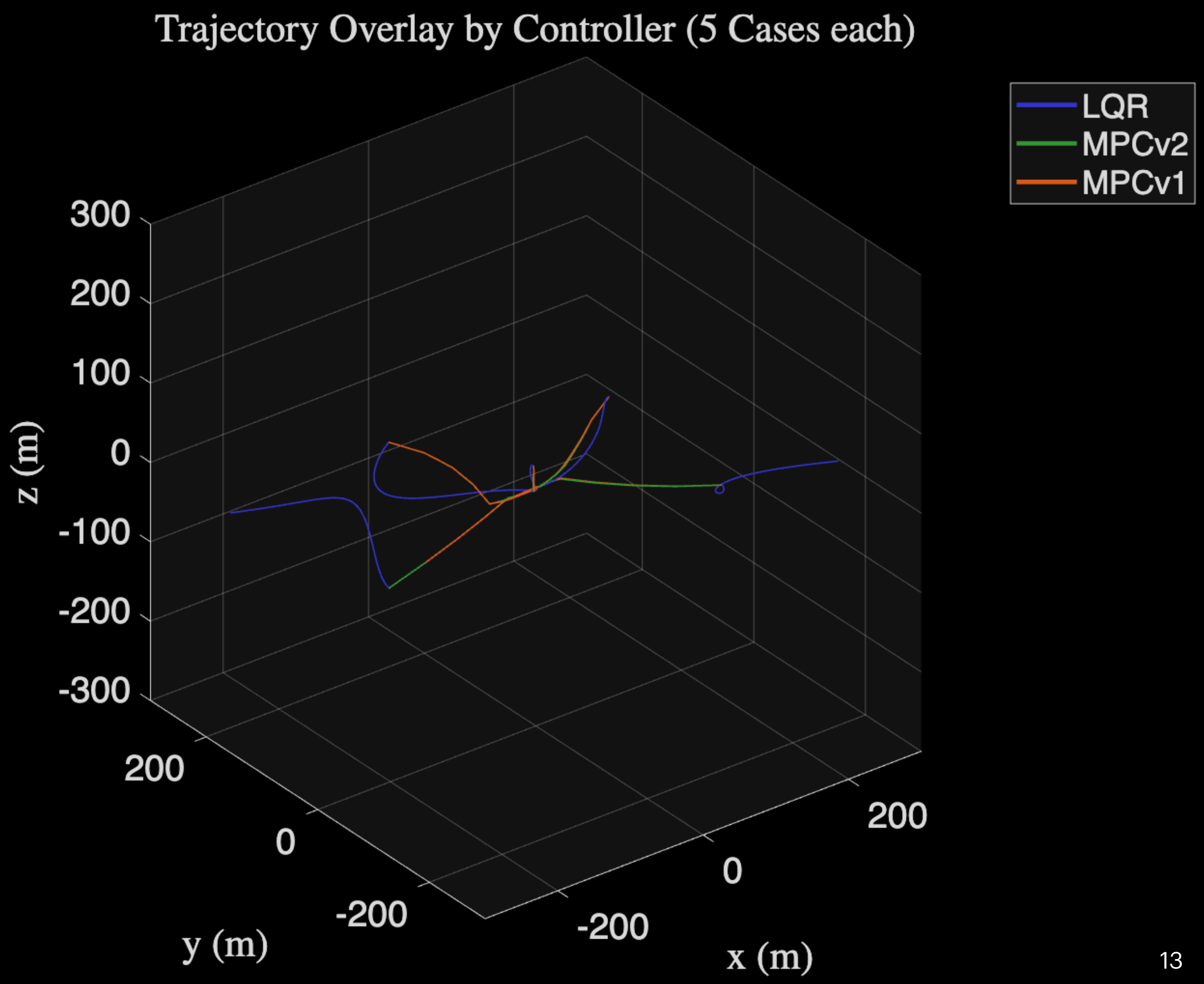
- 50 random initial conditions to see performance against LQR
- Use the same initial conditions
- Apply the same input constraint (clipping LQR signal)



Simulation Results

Trajectories & Trade-offs

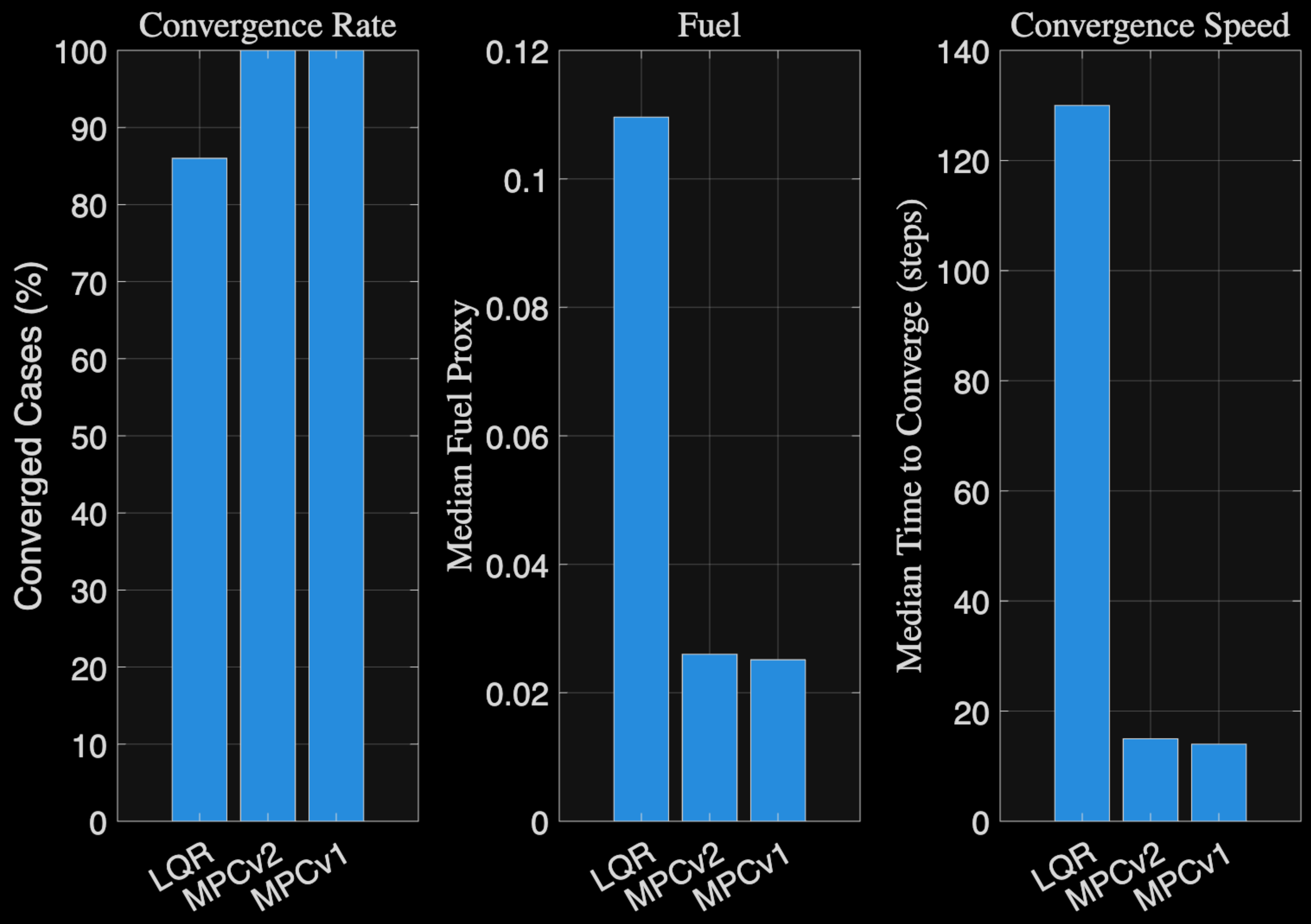
- We pick 5 randomly
- Clearly can see some the LQR controllers driving state unstable
- Tracking and Economic tend to tend to follow very similar trajectories
- Controllers are always stabilizing even if the problem is infeasible at times.



Simulation Results

Trajectories & Trade-offs

- So the LQR controller is able to stabilize quite a lot of initial conditions
- The cost between the two MPC controllers is dwarfed by the LQR
- When it does stabilize, the LQR takes a lot more time to converge to the equilibrium compared to MPC

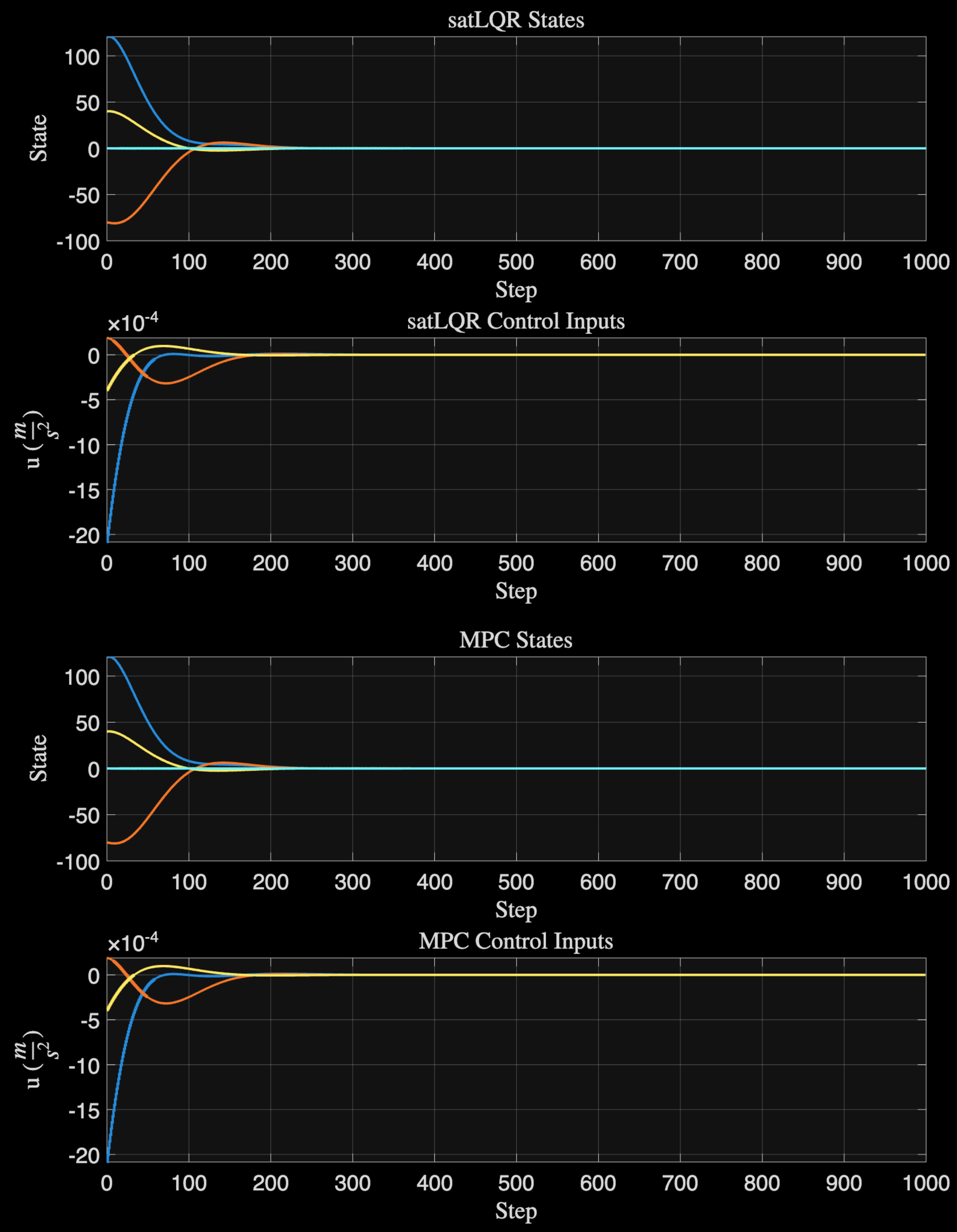


"Fair" Comparison

Trajectories & Trade-offs

- Take the terminal cost to be $\mathbf{diag}(P_f)$ from dlqr
- Use horizon >> time to settle
- Use Q and R with R >> Q s.t. LQR is stabilizing with constraints
- Then simulate without constraints

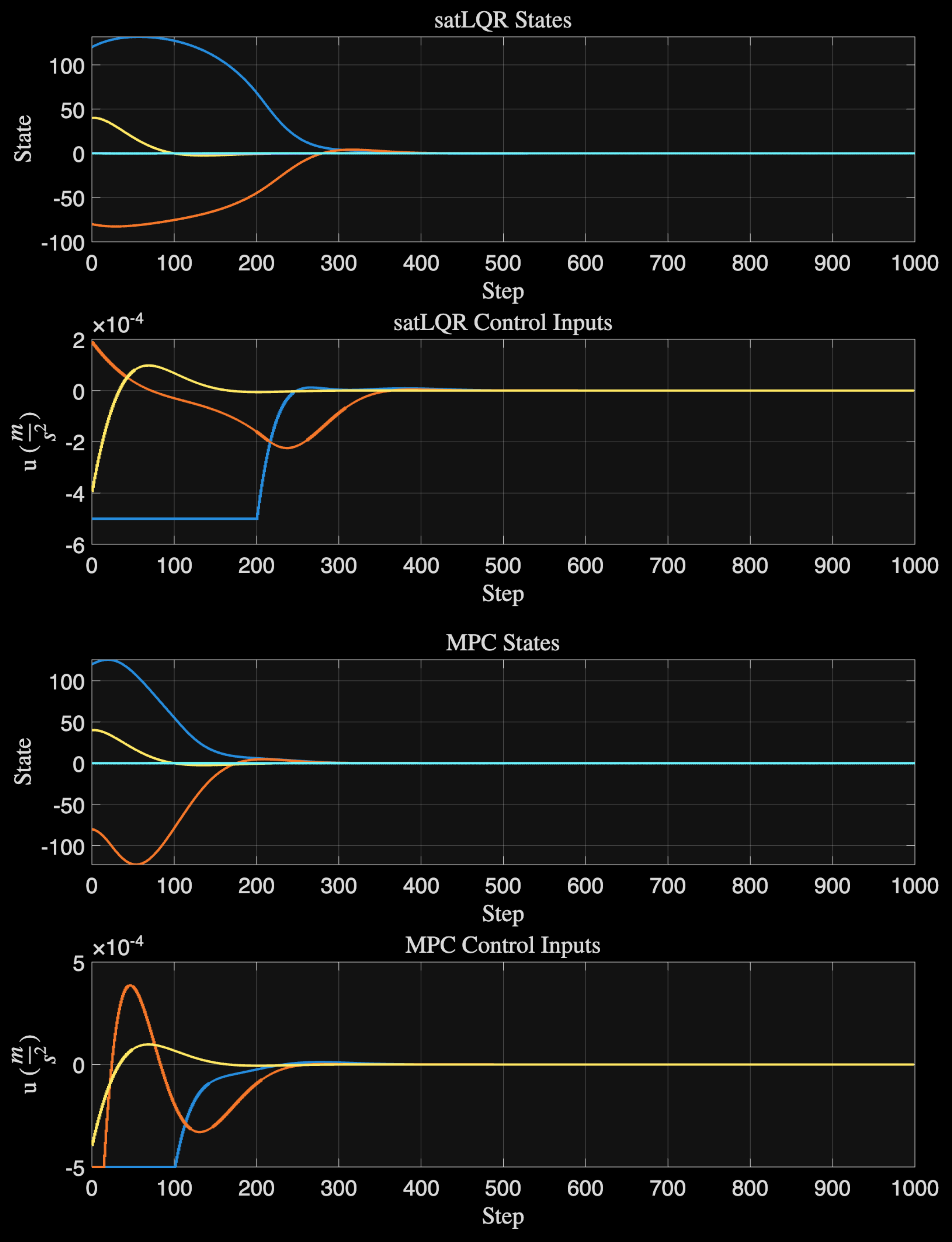
- **Clearly they are the same**



“Fair” Comparison

Trajectories & Trade-offs

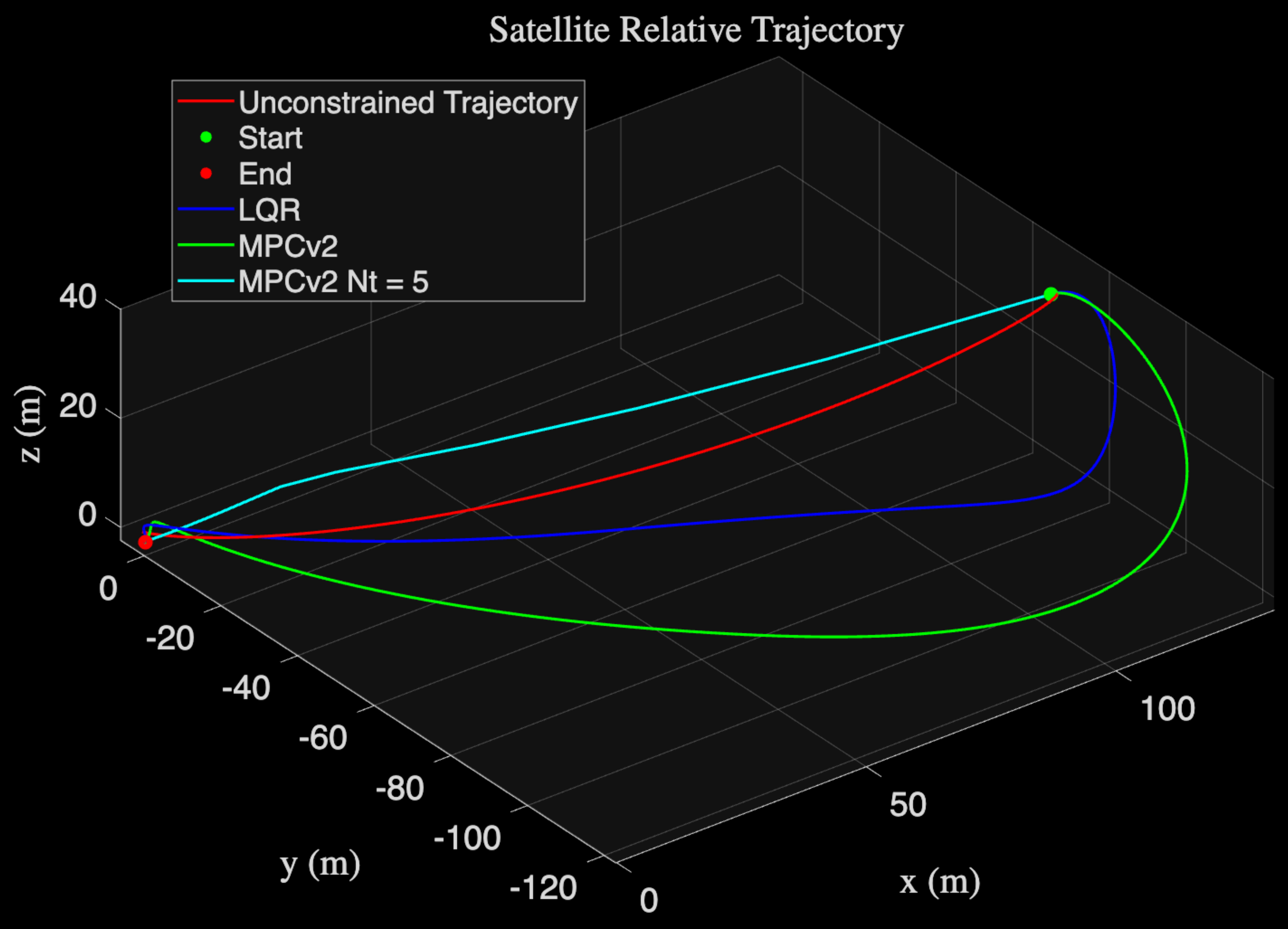
- **Now just add the constraint**
  - MPCv2 fuel use → (sum |u|): 0.129735
  - LQR fuel use → (sum |u|): 0.156989
  - MPCv2 lets states grow larger but uses less fuel with the constraint



"Fair" Comparison

Trajectories & Trade-offs

- **Now just add the constraint**
  - MPCv2 fuel use → (sum |u|): 0.129735
  - LQR fuel use → (sum |u|): 0.156989
  - MPCv2 lets states grow larger but uses less fuel with the constraint
- And allowing infeasibility with Nt = 5 gives **5x decrease in fuel** use to 0.0229219
  - (The signal looked more like the economic MPC from before)



Discussion

Insights & Baseline

- Successful rendezvous with terminal constraint
- Questions: horizon feasibility
  - Hard terminal sets improve targeting but can cause intermittent infeasibility
  - Even when the solver detects infeasibility, we continue to enter a feasible set and converge
- Compared to simple LQR… I would probably never go back to a linear system with constraints if I have freedom to run MPC

References

- [1] X. Lin, G. Zhang and R. Vazquez, "Spacecraft Constant-Thrust Minimum-Time Rendezvous via Reachable Set Theory," in IEEE Transactions on Aerospace and Electronic Systems, vol. 61, no. 5, pp. 13224-13232, Oct. 2025, doi: 10.1109/TAES.2025.3576316.

- [2] "Clohessy–Wiltshire equations," Wikipedia, https://en.wikipedia.org/wiki/Clohessy%E2%80%93Wiltshire_equations, accessed Mar. 9, 2026.

- [3] W. H. Clohessy and R. S. Wiltshire, "Terminal Guidance System for Satellite Rendezvous," Journal of the Aerospace Sciences, vol. 27, no. 9, pp. 653–658, 1960.

- [4] J. B. Rawlings, D. Q. Mayne, and M. M. Diehl, Model Predictive Control: Theory, Computation, and Design, 2nd ed., Nob Hill Publishing, 2017.

- [5] J. B. Rawlings Group, Risbeck, M.J., Rawlings, J.B., 2016. MPCTools: Nonlinear model predictive control tools for CasADi (Octave interface).